TIMES MEASUREMENTS BASED ON QUANTUM PHENOMENA: LIFETIME OF A QUANTUM LEVEL

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Abstract. *The end of the twentieth century saw a great change of the ideas about the units and the standards of measurement. The system based on mechanical standards such as the standard meter of Sevres is replaced by another system based on atomic phenomena, thus quantum, such as a wavelength of light. The new system is now internationally admitted by definitions of the meter based over a wavelength of light of krypton and second based on a hyperfine transition from cesium. The advantages of the new system are partly of conceptual nature, but mainly they are definitely more practical since the new standards are more generally available and allow more precise and more convenient measurements in science and technology. The possibilities of the new approach to metrology go much further those measures of length and time. One will present the methods of calculating of the life time associated with an electron in quantum states and electronic quantum transitions between the levels. For the knowledge of the electronic structure of atoms and ions the gas phase is the state which presents the least difficulty. For this physical state, the parameter to be calculated is chiefly the radiation force S. The adjustment of gaseous lasers, for instance, requires rather the values of the transition probability calculated per unit time. For the radiation force and for the probability of transition the introduction of a dimensionless parameter called oscillator force is expedient. It carries information leading to the comparison between the transition of absorption and emission within the same electronic structure. This evaluation is less easy in the condensed state be it solid or liquid. Here the notion of the oscillator force is essential. For the solid state of matter, by the use of the specter an "experimental" force of oscillation can be estimated. Moreover the symmetry of the site within the crystal must be well known, so as to be able to take into account the effects of the crystalline field.*

Key words: force of oscillator; radiation force;"3j" symbols; C^k coefficients; Clebsch-Gordan coefficients; Einstein coefficients; transition probability by time unit.

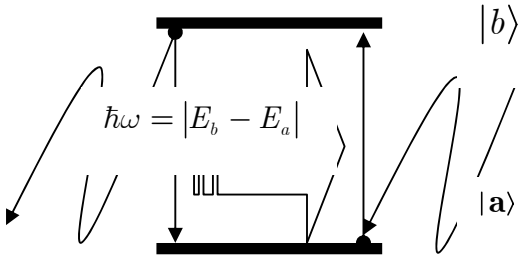
Rezumat. *Sfârşitul secolului douăzeci a fost martorul unor schimbări majore în ce priveşte unităţile de măsură şi etaloanele. Sistemul bazat pe etaloane mecanice cum ar fi metrul etalon de la Sèvres este înlocuit cu un alt sistem bazat pe fenomene atomice, deci cuantic, cum ar fi lungimea de undă a radiaţiei luminoase. Noul sistem este în prezent recunoscut internaţional prin definiţiile pentru metru, bazată pe lungimea de undă a radiaţiei kryptonului, şi pentru secundă, bazată pe o tranziţie hiperfină a cesiului. Avantajele noului sistem sunt în parte de natură conceptuală dar ce este mai important este că aceste definiţii sunt în mod clar mai practice pentru că noile etaloane sunt mai larg accesibile şi permit măsurări mai convenabile pentru ştiinţă şi tehnologie. Posibilităţile noii abordări din metrologie merg însă mai departe decât aceste măsurări de lungimi şi de timp. Se vor astfel prezenta metode de calcul pentru timpul de viaţă asociat unui electron aflat într-o stare cuantică şi tranziţii electronice cuantice între două nivele.Pentru cunoaşterea structurii electronice a atomilor şi ionilor, faza gazoasă este cea care prezintă cea mai mica dificultate. Pentru aceasta stare fizică, parametrul care trebuie în primul rând calculat este forţa de radiaţie S. Reglarea laserilor gazoşi, de exemplu, necesită în primul rând valorile probabilităţii de tranziţie calculată pentru unitatea de timp. Pentru forţa de radiaţie şi pentru probabilitatea de tranziţie este avantajoasă introducerea unui parametru adimensional numit forţă a oscilatorului. Acesta conţine informaţii care conduc la compararea dintre tranziţia de absorbţie şi de emisie în cadrul aceleiaşi structuri electronice. Această evaluare nu mai este la fel de uşoară în starea condensată fie aceasta solidă sau lichidă. În cazul acesta noţiunea de forţă a oscilatorului este esenţială. Pentru starea solidă de agregare, folosind spectrul, se poate estima o forţă "experimentală" de oscilaţie. Mai mult, simetria locului din cristal trebuie să fie binecunoscută, astfel încât să poată fi luate în considerare efectele câmpului cristalin.*

Cuvinte cheie: forţă a oscilatorului; forţă de radiaţie; simboluri "3j"; coeficienţi C^k; coeficienţi Clebsch-Gordan; coeficienţi Einstein; probabilitate de tranziţie în unitatea de timp.

TRANSITION 1s <-> 2p OF THE HYDROGEN ATOM. FORCE OF OSCILLATOR AND RADIATION FORCE

Let us choose a simple case: the transition 1s <-> 2p of the Hydrogen atom (considered as a very rarified gaseous system). This transition is observed at 1216,68 Å (Å = Angstrom).

A quantum transition (absorption or emission) between two electronic states is represented by the following scheme:



The force of oscillator $f_{(ba)}$ of the transition in absorption of $|a\rangle$ toward $|b\rangle$ is bound to the $S_{(ba)}$ "radiation force" through the relation:

$$f_{(ba)} = \frac{2m_0 \cdot \omega_{b\leftrightarrow a}}{3\hbar \cdot e_0^2 \cdot g_a} S_{(ba)} .$$

In this relation m_0 is the true mass of the electron, $\hbar$ is the rationalized Planck constant (h divided by 2π), and e_0 is the normalized charge of the electron (elementary charge of the electron divided by $\sqrt{4\pi\varepsilon_0}$).

$S_{(ba)}$ does not depend from the direction that the transition takes, i.e. absorption or emission:

$$S_{(ba)} = S_{(ab)} = e_0^2 \cdot \sum_{M_a M_b} \left| \langle \alpha_b J_b M_b | R | \alpha_a J_a M_a \rangle \right|^2 .$$

α_a and α_b are supplementary quantum numbers which describe more accurately the states $|a\rangle$ and $|b\rangle$; J_a, J_b and M_a, M_b are respectively the total angular moments and the quantum numbers of "projection" connected with J.

R is here an operator which represents the position vector and corresponds to the median distance of the electron "i" (the one which causes the transition) to the nucleus. For an atom with N electrons we can write R = $\hat{R} = \frac{1}{N} \cdot \sum_{i=1}^{N} \hat{r}_i$. For the Hydrogen atom with only one electron R = r (represented by $\hat{r}$).

In an absorption of level $|a\rangle$ toward the level $|b\rangle$, the radiation force is bound to the Einstein coefficient the following relation:

$$B_{(ba)} = \frac{4\pi^2}{3\hbar^2 \cdot g_a} \cdot S_{(ba)}$$

In an emission from level $|b\rangle$ to the level $|a\rangle$ the radiation force $S_{(ba)}$ and the Einstein coefficient $A_{(ab)}$ are related by:

$$A_{(ab)} = \frac{4\omega^3}{3\hbar \cdot c^3 \cdot g_b} \cdot S_{(ab)} ; \; c \approx 3 \cdot 10^8 \text{ m} \cdot \text{s}^{-1}.$$

Since $\omega = \frac{2\pi \cdot c}{\lambda}$; then:

$$A_{(ab)} = \frac{32\pi^3}{3\hbar \cdot \lambda^3 \cdot g_b} \cdot S_{(ab)} .$$

The units S.I. – u.s.i. (S.I. = international units) of the radiation force $S_{(ab)}$ are the units of the product $e_0^2 \times a_0^2$. In the S.I. system the factor e_0^2 represents the squared elementary charge (e^2) divided by $4\pi\varepsilon_0$ ($\frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9$ $F^{-1} \cdot m$). The calculation of the element of the matrix involves as well the square of the radius of the first Bohr orbit a_0. Therefore with the usual values for the elementary charge and the radius of the first Bohr orbit, we can calculate:

$$\left. \begin{array}{l} e \approx 1{,}6 \cdot 10^{-19} C \; ; \; e_0 = \dfrac{e}{\sqrt{4\pi\varepsilon_0}} \\ a_0 \approx 0{,}529 \, \dot{A} = 0{,}529 \cdot 10^{-10} m \end{array} \right\} \Rightarrow e_0^{2\cdot} \times a_0^2 \approx$$

$$\approx 0{,}644 \cdot 10^{-48} \, u.s.i.$$

(Joule × meter^3).

If we wish to express λ in Å, knowing that $\hbar \approx \approx 1{,}054 \times 10^{-34}$ u.s.i., the Einstein coefficient must be written in the form:

$$A_{(ab)} = \frac{2{,}02 \times 10^{18}}{\lambda^3_{\left(\overset{\circ}{A}\right)}} \cdot g_b \cdot S_{(ab)} .$$

This coefficient has a dimension, which corresponds to the inverse of a time (s^{-1}).

The calculation of the radiation force $S_{(ab)}$ for the Hydrogen atom gives:

$$(R \rightarrow \hat{r}) \Rightarrow$$

$$\Rightarrow S_{(ab)} = e_0^2 \cdot \sum_{M_a M_b} \left\| \langle \alpha_b J_b M_b | \hat{r} | \alpha_a J_a M_a \rangle \right\|^2 .$$

For this type of calculation the vector r is easier to express on the basis of spherical harmonics i.e.:

$r_0 = z = \sqrt{\frac{4\pi}{3}} \cdot \|r\| \cdot Y_{10}$ and

$r_{\pm 1} = \mp \frac{1}{\sqrt{2}} (x \pm iy) = \; = \sqrt{\frac{4\pi}{3}} \cdot \|\mathbf{r}\| \cdot Y_{1\pm 1}$. In the following calculations r is usually written: $\|\mathbf{r}\| = r$.

In the case of emission this calculation is written by the use of the single-electronic states Φ_i:

$$\left\| \langle \alpha_b J_b M_b | \hat{r} | \alpha_a J_a M_a \rangle \right\|^2 = \left\| \langle \Phi_{2p} | \hat{r} | \Phi_{1s} \rangle \right\|^2 ;$$

$$|\Phi_i\rangle = \frac{1}{r} \cdot P_{nl}(r) \cdot Y_{lm}(\theta\varphi) = R_{nl} \cdot Y_{lm}(\theta\varphi) .$$

A single-electronic state Φ_i is expressed as the product between a radial factor and an angular factor depending from θ and from φ. The vector **r** is expressed in function of $\|\mathbf{r}\|.Y_{LM}$.

Let us consider the quantity $\langle n'\, l'\, m'|$ for the function 2p and $\langle n\ l\ m|$ for the function 1s. The separation of the radial parts from the angular parts then given:

$$\langle \Phi_{2p}|\hat{r}|\Phi_{1s}\rangle = \langle n'l'm'|\hat{r}|nlm\rangle = \langle P_{2p}|r|P_{1s}\rangle \cdot \langle l'm'|Y_{LM}|lm\rangle .$$

Considering first the radial parts:

$$P_{2p}(r) = \frac{1}{2\sqrt{6}} \cdot \frac{r^2}{a_0^{5/2}} \cdot \exp\left(-\frac{r}{2a_0}\right);$$

$$P_{1s}(r) = \frac{2r}{a_0^{3/2}} \cdot \exp\left(-\frac{r}{a_0}\right)$$

$$\langle P_{2p}|r|P_{1s}\rangle =$$

$$= \int_0^\infty \frac{1}{2\sqrt{6}} \cdot \frac{r^2}{a_0^{5/2}} \cdot \exp\left(-\frac{r}{2a_0}\right) \cdot r \cdot \frac{2r}{a_0^{3/2}} \cdot \exp\left(-\frac{r}{a_0}\right) dr =$$

$$= \frac{1}{\sqrt{6}} \cdot \frac{1}{a_0^4} \int_0^\infty r^4 \cdot \exp\left(-\frac{3r}{2a_0}\right) \cdot dr .$$

The integrals of the kind $\int_0^\infty x^n \cdot \exp(-\alpha x)\, dx = \frac{n!}{\alpha^{n+1}}$ were used above.

In the case considered above n = 4 and $\alpha = \frac{3}{2 \cdot a_0}$; so the calculation gives: $\frac{4!}{\sqrt{6}} \cdot \frac{1}{a_0^4} \cdot \left(\frac{2}{3}\right)^5 \cdot a_0^5 \approx \sqrt{\frac{5}{3}} \cdot a_0$. Therefore in the calculation of the radiation force, the squared norm of the radial part is: $\left\|\langle P_{2p}|r|P_{1s}\rangle\right\|^2 \approx \frac{5}{3} \cdot a_0^2$.

For the angular parts, the calculation is:

$$\langle l'm'|Y_{LM}|lm\rangle = \int_0^\pi \sin\theta \cdot d\theta \int_0^{2\pi} Y^*_{l'm'} \cdot Y_{LM} \cdot Y_{lm}\, d\varphi =$$

$$= \iint_{\theta\in[0,\pi];\varphi\in[0,2\pi]} Y^*_{l'm'} \cdot Y_{LM} \cdot Y_{lm}\, d\Omega ;\ d\Omega = \sin\theta\ d\theta d\varphi .$$

The angular integrals are calculated in several different ways: with the "3j" symbols; with the coefficients C^k or with the Clebsch-Gordan coefficients.

"3j" SYMBOLS

The "3j" symbols (see Rotenberg's tables) correspond to the coupling of 3 angular moments (two moments in relation to the third)

$$\langle l'm'|Y_{LM}|lm\rangle =$$

$$= (-1)^{m'} \sqrt{\frac{(2l'+1)\cdot(2L+1)\cdot(2l+1)}{4\pi}} \cdot \begin{pmatrix} l' & L & l \\ -m' & M & m \end{pmatrix} \cdot \begin{pmatrix} l' & L & l \\ 0 & 0 & 0 \end{pmatrix}.$$

The calculations yield results different from zero, if the rule of triangulation is satisfied. First - m' + M + m must be 0, therefore M = = m' − m and secondly the sum l' + L + l must be one even whole number. For a dipolar electrical transition, the expression of "r" in spherical harmonics, L=1 is obligatory.

C^k COEFFICIENTS

The C^k coefficients correspond directly to the integrals of our research:

$$C^k(lm;l'm') = \sqrt{\frac{4\pi}{2k+1}} \langle Y_{lm}|Y_{km-m'}|Y_{l'm'}\rangle .$$

Another condition is that: $C^k(lm;l'm') = (-1)^{m-m'} C^k(l'm';lm)$.

For the spherical of harmonics Φ_{2p} we have l' = 1 and m' = + 1, 0, - 1. The Y_{lm} to be considered are therefore Y_{11}, Y_{10} and Y_{1-1}.

For Φ_{1s} : l = 0 and m = 0, therefore the only possibility for Y_{lm} is Y_{00}.

In applying the rules of triangulation:

$$\sqrt{\frac{4\pi}{3}} \langle Y_{11}|Y_{11}|Y_{00}\rangle = C^0(11;00) = \frac{1}{\sqrt{3}};$$

$$\sqrt{\frac{4\pi}{3}} \langle Y_{10}|Y_{10}|Y_{00}\rangle = C^0(10;00) = \frac{1}{\sqrt{3}}$$

and $\sqrt{\frac{4\pi}{3}} \langle Y_{1-1}|Y_{1-1}|Y_{00}\rangle = C^0(1-1;00) = \frac{1}{\sqrt{3}}$.

Then: $\left(\sqrt{\frac{4\pi}{3}} \langle l'm'|Y_{LM}|Y_{00}\rangle\right)^2 = \frac{1}{3}$.

Finally, the calculation of the value of $(r)^2$ between 1s and 2p yields:

$$\left\|\langle \Phi_{2p}|r|\Phi_{1s}\rangle\right\|^2 =$$

$$= \left\|\langle P_{2p}|r|P_{1s}\rangle\right\|^2 \cdot \left\|\langle l'm'|Y_{LM}|lm\rangle\right\|^2 \approx \frac{5}{9} a_0^2 .$$

For the radial parts, the integral intervenes for one value only. Each one of the three integrals for the angular parts represents one possibility of absorption, according to the polarization of the photons (according to x, y, or z).

This is the reason for using the symbol $\sum_{M_a M_b}$.

Let us recall that:

$$S_{(ab)} = e_0^2 \cdot \sum_{M_a M_b} \left\| \left\langle \alpha_b J_b M_b \left| \hat{r} \right| \alpha_a J_a M_a \right\rangle \right\|^2 .$$

The sum of these three possibilities corresponding to the three integrals is 1/3+1/3+1/3= = 1.

The radiation force of the transition 1s → 2p is thus obtained:

$$S_{(1s\to 2p)} = e_0^2 \cdot \sum \left\| \left\langle \Phi_{2p} \left| \hat{r} \right| \Phi_{1s} \right\rangle \right\|^2 = \frac{5}{3} \cdot e_0^2 \cdot a_0^2 .$$

CLEBSCH-GORDAN COEFFICIENTS

The Clebsch-Gordan coefficients are related the symbols "3j" by the following relation:

$$\frac{(-1)^{j_1 - j_2 - m}}{\sqrt{2j+1}} \cdot \left\langle j_1 j_2 m_1 m_2 \left| j_1 j_2 j \;\; -m \right\rangle = \right.$$

$$= \begin{pmatrix} j_1 & j_2 & j \\ m_1 & m_2 & m \end{pmatrix} .$$

The "j" and "m" act as j_3 and m_3 in the general expression of the symbol "3j". The tables of the Clebsch-Gordan coefficients show:

$$\langle 11-1-1|1100\rangle = \frac{1}{\sqrt{3}} .$$

Therefore the first symbol "3j" calculated above could be obtained also by the use of the Clebsch-Gordan coefficients:

$$\frac{(-1)^{1-1-0}}{\sqrt{2\cdot 0+1}} \cdot \langle 11-11|1100\rangle = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \end{pmatrix} = .$$

$$= \frac{(-1)^0}{\sqrt{1}} \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

EINSTEIN COEFFICIENTS

In an emission type transition b → a the radiatif lifetime τ of the starting level b is defined as the inverse of the Einstein coefficient $A_{(ab)}$. The transition 1s ↔ 2p of Hydrogen la observed at λ = 1216,68 Å and the initial level 2p, as p type orbital has a degeneration which value is 3; therefore $g_{2p} = 3$.

In this case the Einstein coefficient at the spontaneous emission can be calculated:

$$A_{(1s,2p)} = \frac{2{,}02 \cdot 10^{18} \cdot S_{(a,b)}}{\lambda^3 \left(\dot{A}\right) g_{2p}} \approx 6{,}25 \cdot 10^8 \, s^{-1} .$$

From the coefficient $A_{(ls\ 2p)}$, we obtain the lifetime of the level 2p :]

$$\tau = \left(A_{(1s,2p)}\right)^{-1} \approx 0{,}16 \cdot 10^{-8} \text{s}.$$

From the knowledge of the radiation force $S_{(ls\to 2p)} = S_{(2p\to 1s)}$ the force of oscillation at the absorption $f_{(2p1s)}$ can be obtained directly. For this, the same method will be used as previously in expressing the constant factors of the general formula in the S.I. system, retaining only the variable λ and g in the final expression. Taking into account the product $e_0^2 \times a_0^2$ (unit of $S_{(ba)}$), we obtain:

$$f_{(ba)} = \frac{2m_0 \cdot 2\pi\nu}{3\hbar \cdot e_0^2 \cdot g_a} \cdot S_{(ba)} = \frac{2m_0 \cdot 2\pi \dfrac{c}{\lambda}}{3\hbar \cdot e_0^2 \cdot g_a} \cdot S_{(ba)} \approx$$

$$\approx \frac{303{,}7}{\lambda\left(\dot{A}\right) \cdot g_a} \cdot S_{(ba)}$$

$$\left(\text{in } e_0^2 \cdot a_0^2 \text{ units}\right).$$

For the Hydrogen atom, this corresponds to the absorption of level ls towards 2p:

$$f_{(2p1s)} = \frac{2m_0 \cdot 2\pi\nu_{2p\leftarrow 1s}}{3\hbar \cdot e_0^2 \cdot g_{1s}} \cdot \frac{5}{3} =$$

$$= \frac{2m_0 \cdot 2\pi \dfrac{c}{\lambda_{2p\leftarrow 1s}}}{3\hbar \cdot e_0^2 \cdot g_{1s}} \cdot \frac{5}{3} \approx \frac{303{,}7}{\lambda_{2p\leftarrow 1s}\left(\dot{A}\right) \cdot g_{1s}} \cdot \frac{5}{3} .$$

In the transition 1s→2p, as the 1s level is not degenerated, we have g_{1s}=1 and $\lambda_{2p\leftarrow 1s}$ = = 1216,68 Å.

Under these conditions at the absorption the oscillator force $f_{(2p1s)}$ equals ≈ 0,416.

That the force of oscillator "f" has no dimension is then easily verified. From this

value the force of oscillator at emission can also be extracted by the use of the relation:

$$f_{(1s2p)} = -\frac{g_{1s}}{g_{2p}} \cdot f_{(2p1s)} \approx -\frac{1}{3} \cdot 0,416 \approx -0,139 .$$

The emission is on the sign contrary to the absorption that is considered positive.

The radiation force enables also to calculate the Einstein coefficient in absorption:

$$B_{(ba)} = \frac{4\pi^2}{3\hbar^2 \cdot g_a} \cdot S_{(ba)} .$$

Therefore for the transition ls → 2p the Einstein coefficients is:

$$B_{(2p1s)} = \frac{4\pi^2}{3\left(1,054 \cdot 10^{-34}\right)^2 \cdot 1} \cdot S_{(2p1s)} \Rightarrow$$

$$B_{(2p1s)} \approx \frac{4\pi^2}{3 \cdot 1,1 \cdot 10^{-68}} \cdot \frac{5}{3} \cdot 0,644 \cdot 10^{-48} \approx$$

$$1,26 \cdot 10^{21} \quad m^3 \cdot J^{-1} \cdot s^{-2} .$$

The coefficient of spontaneous emission is expressed in second^{-1}. The coefficient characteristic of absorption is in $m^3 \times J^{-1} \times s^{-2}$. In fact the very notion of absorption implies that the system on which the experiment is carried out is subjected to a photons sheaf of monochromatic light.

This sheaf yields at the level of the sample a constant stream of photons of a fixed frequency. These conditions led us to introduce the quantity $U_\omega d\omega$ (spectral density of energy).

$U_\omega d\omega$ represents the energy of n photons (individual energy $\varepsilon = \hbar\omega$) by unit of volume and in the intervals from ω to ω+dω:

$$U_\omega = n \cdot \frac{\hbar \cdot \omega^3}{\pi^2 \cdot c^3} = n \cdot \frac{2\hbar}{\pi \cdot \lambda^3} .$$

The energy by volume unit is sometimes defined in frequency ν and not in pulsation ω. As $\omega = 2\pi\nu$ and $U\omega\, d\omega = U\nu\, d\nu$ the expressions Uω and Uν are related by $U\omega = U\nu/2\pi$. The expressions that bind the coefficients are:

$$B_{(ba)} = \frac{\pi^2 \cdot c^3}{\hbar \cdot \omega^3} \cdot A_{(ba)} .$$

Therefore by returning to the previous calculation : $U_\omega = n \cdot \dfrac{0,67 \cdot 10^{-4}}{\lambda^3\left(\dot{A}\right)} \quad J \cdot s \cdot m^{-3}$.

For the dimensions, the product corresponds then:

$$\left[B_{(ba)} \cdot U_\omega\right]_{\dim} = \left\langle m^3 \cdot J \cdot s^{-2} \right\rangle \cdot \left\langle J \cdot s \cdot m^{-3} \right\rangle = .$$
$$= \left\langle s^{-1} \right\rangle$$

The product $B_{(ba)} \times U_\omega$ has as $A_{(ab)}$ the dimension of the inverse of time.

For the Hydrogen atom with λ = 1216,18 Å the calculations lead to:

$$U_\omega = n \cdot \frac{0,67 \cdot 10^{-4}}{\left(1,21618 \cdot 10^3\right)^3} \approx .$$
$$\approx n \cdot 3;7 \cdot 10^{-14} \quad J \cdot s \cdot m^{-3}$$

The Einstein coefficient of stimulated absorption × U_ω gives:

$$B_{(2p1s)} \cdot U_\omega \approx 1,26 \cdot 10^{21} \quad \cdot \quad n \cdot 3,7 \cdot 10^{-14} \approx .$$
$$\approx n \cdot 4,66 \cdot 10^7 \quad s^{-1}$$

Without special difficulty the same value of the force of oscillation in absorption is obtained, it the relation that expresses this force as a function of the Einstein coefficient is used.

For tile transition 1s → 2p with λ = = 1216,18 Å, it gives $f_{(2p,\ 1s)} = 0,415$.

The "details" of the calculation are as follows:

$$f_{(2p,1s)} = \frac{m_0 \cdot \hbar \cdot \omega_{2p \leftarrow 1s}}{2\pi^2 \cdot e_0^2} \cdot B_{(2p1s)} =$$

$$= \frac{m_0 \cdot \hbar \cdot 2\pi \cdot c}{2\pi^2 \cdot \dfrac{e^2}{4\pi \cdot \varepsilon_0} \cdot \lambda_{2p \leftarrow 1s}} \cdot B_{(2p1s)}$$

$$f_{(2p,1s)} \approx \frac{\left(9,1 \cdot 10^{-31}\right) \cdot \left(1,054 \cdot 10^{-34}\right) \cdot 2 \cdot \left(3 \cdot 10^8\right)}{2(3,14) \cdot \left(1,6 \cdot 10^{-19}\right)^2 \cdot \left(9 \cdot 10^9\right) \cdot \lambda_{\left(\dot{A}\right)} \cdot \left(10^{-10}\right)} \cdot$$

$$\cdot \left(1,26 \cdot 10^{21}\right)$$

$$f_{(2p,1s)} \approx \frac{505}{\lambda_{\left(\dot{A}\right)}} = \frac{505}{1216,18} \approx 0,415 .$$

As for the forces of oscillators, the level's degenerate relates the Einstein coefficients to each other. The Einstein coefficients can then be calculated in stimulated emission by the knowledge of the one in absorption. Thus:

$]g_a \times B_{(ba)} = g_b \times B_{(ab)}$. In the case under study, this gives:

$$g_{1s} \times B_{(2p,\,1s)} = g_{2p} \times B_{(ls,\,2p)} \Rightarrow$$

$$\Rightarrow 1 \times 1{,}26 \times 10^{21} = 3 \times B_{(ls,\,2p)}.$$

Finally we obtain $B_{(ls,\,2p)} = 0{,}42.\ 10^{21}\ m^3 \times \times J^{-1} \times s^{-2}$.

PROBABILITY OF TRANSITION BY TIME UNIT

Beside the forces of oscillators and the Einstein coefficient, we have a third method of writing the equation of these phenomena. It consists in formulating them in term of probability of transition by time unit. This method is easier when a balance sheet is required. This balance sheet is drawn between the energy that enters in the system at absorption and that leaves it at emission, no matter whether it takes place in the spontaneous or stimulated form.

The formulae for the probability of spontaneous emission generalize those of the coefficients:

$$W_{spont}^{(a\leftarrow b)} = A_{(a\leftarrow b)} = \frac{4\omega_{a\leftarrow b}^3}{3\hbar \cdot c^3 \cdot g_b} \cdot S_{(ba)}\,.$$

We have therefore:

$$W_{spont}^{(1s\leftarrow 2p)} = A_{(1s\leftarrow 2p)} = \frac{4\omega_{1s\leftarrow 2p}^3}{3\hbar \cdot c^3 \cdot g_{2p}} \cdot S_{(2p1s)} \approx$$

$$\approx\ 6{,}25 \cdot 10^8 \left\langle s^{-1} \right\rangle .$$

Likewise for the probabilities of stimulated absorption and emission:

$$W_{abs}^{(b\leftarrow a)} = W_{stim}^{(b\leftarrow a)} = B_{(ba)} \cdot U_\omega = \frac{4\omega^3 \cdot n}{3\hbar \cdot c^3 \cdot g_a} \cdot S_{(ab)}$$

In fact:

$$W_{abs}^{(2p\leftarrow 1s)} = W_{stim}^{(2p\leftarrow 1s)} = B_{(2p1s)} \cdot U_\omega =$$

$$= n \cdot 4{,}66 \cdot 10^7 \ \left\langle s^{-1} \right\rangle .$$

Let us consider the relations:

$$W_{abs}^{(b\leftarrow a)} \cdot g_a = W_{stim}^{(b\leftarrow a)} \cdot g_a =$$

$$= W_{abs}^{(a\leftarrow b)} \cdot g_b = W_{stim}^{(a\leftarrow b)} \cdot g_b .$$

In the case 1s→2p this gives:

$$W_{abs}^{(2p\leftarrow 1s)} \cdot 1 = W_{stim}^{(2p\leftarrow 1s)} \cdot 1 =$$

$$= W_{abs}^{(1s\leftarrow 2p)} \cdot 3 = W_{stim}^{(1s\leftarrow 2p)} \cdot 3 .$$

$$W_{stim}^{(1s\leftarrow 2p)} = \frac{W_{abs}^{(2p\leftarrow 1s)} \cdot 1}{3} = n \cdot \frac{4{,}66 \cdot 10^7}{3} \approx$$

$$\approx n \cdot 1{,}55 \cdot 10^7 \ \left\langle s^{-1} \right\rangle .$$

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